A phase-space representation of nucleon-nucleon potentials*

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Effective realistic nucleon-nucleon (NN) potentials that do not scatter to high momenta contain momentum dependent contributions. The Argonne potential [1] transformed by means of the Unitary Correlation Operator Method [2], for example, has a quadratic momentum dependence. Interactions arising from the Similarity Renormalization Group (SRG) method [3] show a more complicated momentum dependence, which is, however, not transparent as these potentials are constructed directly in matrix element representation.

To investigate the momentum dependence of NN potentials given by matrix elements, we use the phase-space representation introduced by Kirkwood [4]. In this representation the phase-space distribution $f_{ps}(\vec{r},\vec{p})$ for a density operator ρ and the representation $O_{ps}(\vec{r},\vec{p})$ of an operator $\mathcal{O}$ are given by

$$f_{ps}(\vec{r},\vec{p}) = (2\pi)^{3/2} \left\langle\, \vec{r} \,\middle|\rho\middle|\, \vec{p}\,\right\rangle\left\langle\, \vec{p}\,\middle|\,\vec{r}\,\right\rangle \tag{1a}$$

$$O_{ps}(\vec{r},\vec{p}) = (2\pi)^{3/2} \left\langle\, \vec{r} \,\middle|\mathcal{O}\middle|\, \vec{p}\,\right\rangle\left\langle\, \vec{p}\,\middle|\,\vec{r}\,\right\rangle, \tag{1b}$$

such that

$$\left\langle\, \mathcal{O}\,\right\rangle = \mathrm{Tr}\,(\rho\mathcal{O}) \int \mathrm{d}^3r\mathrm{d}^3p\, f^*_{ps}(\vec{r},\vec{p}) \cdot O_{ps}(\vec{r},\vec{p}). \tag{1c}$$

For a potential given in partial wave matrix elements $\left\langle\, kLM;S;T \,\middle|\mathbf{V}\middle|\, pLM;S;T\,\right\rangle$, with the momentum quantum numbers L and M and spin and isospin S and T, Eq. (1b) can be rewritten as

$$\begin{aligned} V_{ps}(\vec{r},\vec{p}) =& 4\pi e^{-i\vec{r}\vec{p}} \sum_{L,M} i^L Y^L_M(\hat{\vec{r}}) Y^{L\,*}_M(\hat{\vec{p}}) \times \\ & \int_0^\infty \mathrm{d}k k^2 \left\langle\, kLM;S;T \,\middle|\mathbf{V}\middle|\, pLM;S;T\,\right\rangle j_L(rk), \end{aligned} \tag{2}$$

where Y^L_M is a spherical harmonic and j_L a spherical Bessel function. We describe the angular part of $V_{ps}(\vec{r},\vec{p})$ by an expansion in Legendre polynomials $P_\Lambda(\hat{\vec{r}}\cdot\hat{\vec{p}})$:

$$V_{ps}(\vec{r},\vec{p}) = \sum_\Lambda i^\Lambda V^\Lambda_{ps}(r,p) P_\Lambda(\hat{\vec{r}}\cdot\hat{\vec{p}}). \tag{3}$$

Fig. 1 shows the first terms of this expansion calculated by means of Eq. (2) from the matrix elements of different potentials, namely a local potential $V(\mathbf{r})$, a potential with quadratic momentum dependence, and a potential with quadratic angular momentum dependence. For the local potential, $V_{ps}(\vec{r},\vec{p})$ is just $V(r)$ and the phase-space representation does not depend explicitely on p and the angle between $\vec{r}$ and $\vec{p}$. For the quadratic momentum dependent potential $\mathbf{V} = \frac{1}{2}\left(\vec{\mathbf{p}}^{\,2}V(\mathbf{r}) + V(\mathbf{r})\vec{\mathbf{p}}^{\,2}\right)$ the phase-space representation shows a characteristic quadratic momentum dependence for $\Lambda = 0$ and a $\Lambda = 1$ contribution reflecting the fact that $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$ do not commute. All higher Λ-contributions vanish. $V_{ps}(\vec{r},\vec{p})$ of the quadratic angular momentum potential contains terms up to $\Lambda = 2$. Potentials with more complicated momentum dependencies, for example from a SRG transformation, would create contributions also for higher Λ.

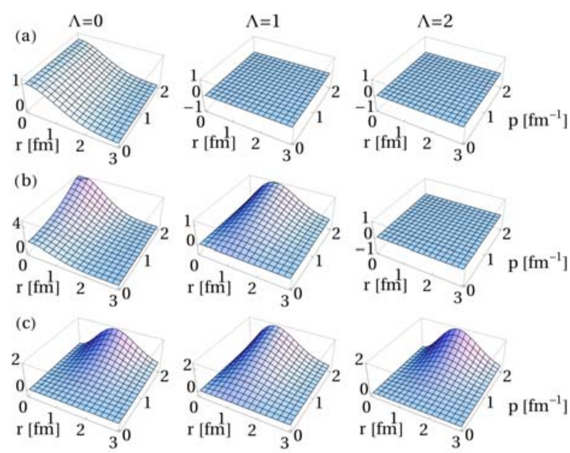

Figure 1: Phase-space representation $V^\Lambda_{ps}(r,p)$ in arbitrary units for (a) $\mathbf{V} = V(\mathbf{r})$, (b) $\mathbf{V} = \frac{1}{2}\left(\vec{\mathbf{p}}^{\,2}V(\mathbf{r}) + V(\mathbf{r})\vec{\mathbf{p}}^{\,2}\right)$, (c) $\mathbf{V} = V(\mathbf{r})\,\vec{\mathbf{L}}^2$. $V(r) = e^{-\frac{r^2}{2\,\mathrm{fm}^2}}$.

These results show that the phase-space representation is able to visualize the (non-) local structure of a potential. In further studies we plan to employ this method to investigate the momentum dependence of various realistic NN potentials given in matrix representation.

References

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* Supported by the Helmholtz Alliance EMMI